APPLIED MECHANISM DESIGN FOR SOCIAL GOOD

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Lecture #23 – 04/18/2022
Lecture #24 – 04/20/2022

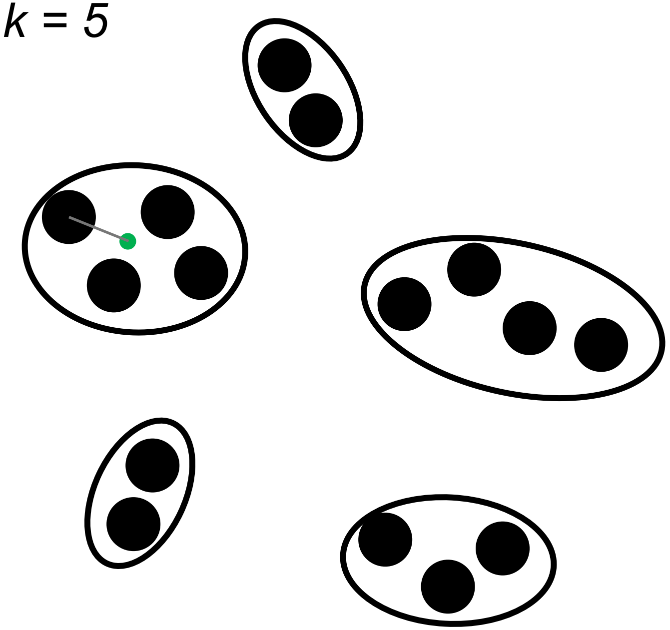
CMSC498T
Mondays & Wednesdays
2:00pm – 3:15pm

COMPUTER SCIENCE
UNIVERSITY OF MARYLAND

CLUSTERING PRIMER

Clustering is the problem of grouping data based off of the distance (or similarity score) between points.

$k = 5$



k-center: minimize the maximum distance from a point to its cluster center.

k-median: minimize the sum of distances from points to cluster centers.

k-means: minimize the sum of squares of distances from points to cluster centers.

CLUSTERING PRIMER

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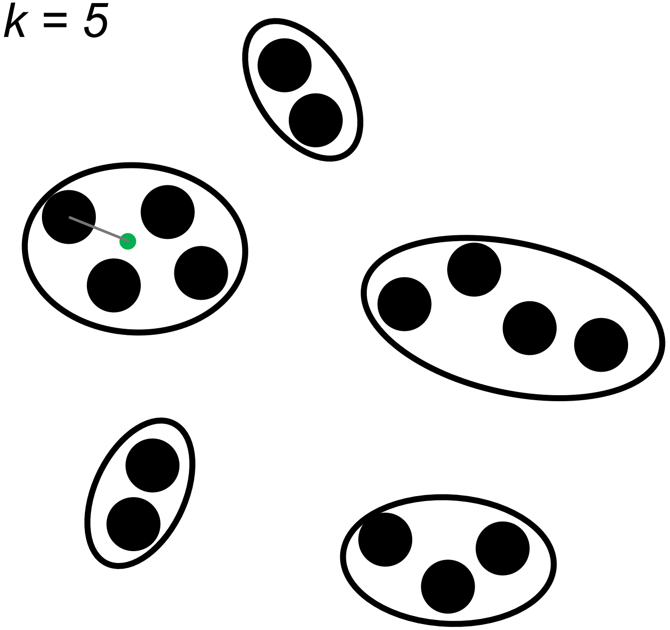


Generally, we minimize:

$$\sum_{x \in X} ||\varphi(x) - x||^p$$

X is our point set.

φ tells us the center for a point.

p defines what we maximize.

k-center: p = ∞

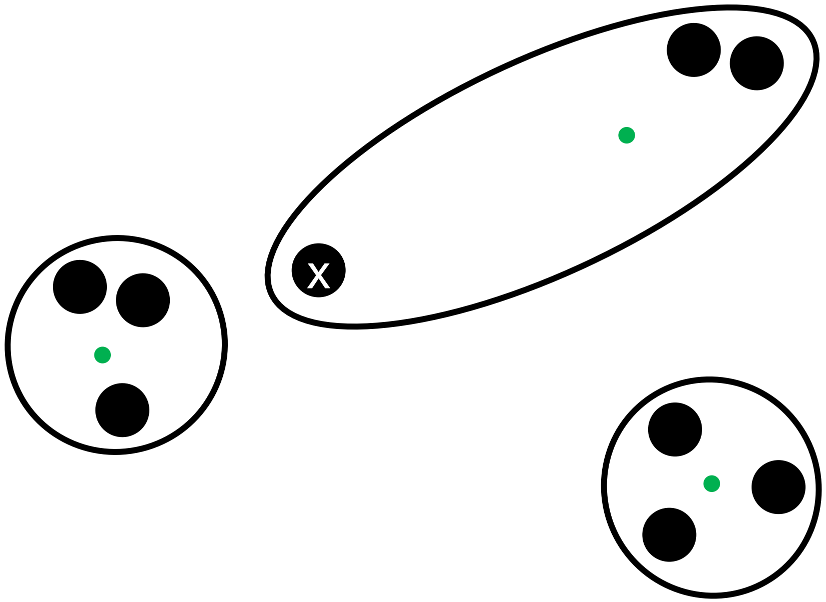
k-median: p = 1

k-center: p = 2

…

CLUSTERING PRIMER

Clustering *x* poorly is only one of many costs in *k-median* since we sum over many points. But for *k-center*, since it is the furthest point from the cluster center, it is very costly, in fact it defines the *k-center* cost.



CLUSTERING PRIMER

We can write an integer linear program to assign points to clusters.

Let $C = c_1, c_2, \dots, c_k$ be the cluster centers.

Let x_i denote that x is assigned to center i.

This may look nonlinear, but it's a constant!

Objective: minimize $\sum_{c_i \in C} \sum_{x \in X} x_i \times \boxed{||c_i - x||^p}$

Constraints: $\sum_{c_i \in C} x_i = 1$ for all $x \in X$ (only assign to 1 center)

$x_i \in \{0,1\}$ for all $x \in X$ and $c_i \in C$

CLUSTERING PRIMER

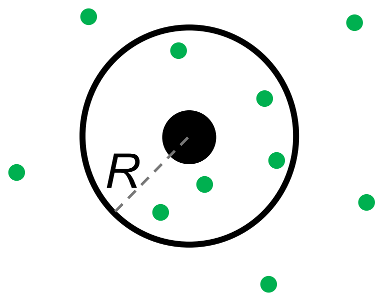
For *k-centers*, we do it a bit differently. We do *not* put anything in the objective. We guess *R* as an upper bound on the distance to centers. *R* is the cost of the solution. We use binary search to find the smallest *R* with a feasible solution.

Objective: None!

Constraints: $\sum_{c_i \in C \cap B_R(x)} x_i = 1$ for all $x \in X$ (only assign to 1 center)

$x_i = 0$ for all $x \in X$ and $c_i \in C \setminus B_R(x)$

$x_i \in \{0,1\}$ for all $x \in X$ and $c_i \in C$



Note: $B_R(x)$ is the ball of radius *R* around *x* (i.e., points within *R* from *x*).

DISPARATE IMPACT

Griggs vs Duke Power Co:

- North Carolina, 1970
- Required high school diploma and standardized testing for promotion
- Sued for discrimination
- Ruled discriminatory by SC because it had “a disproportionate and adverse impact on certain individuals”

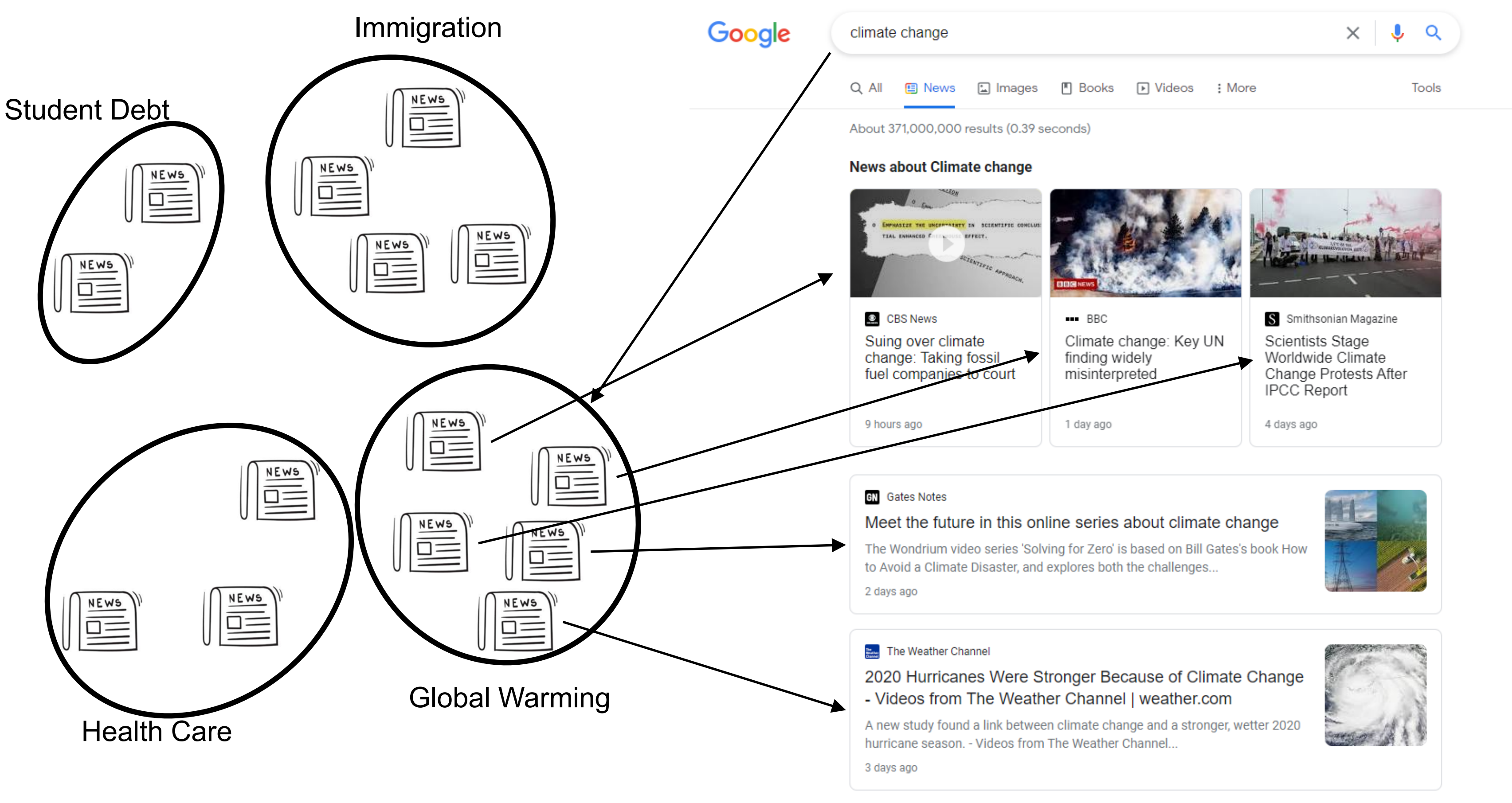
DISPARATE IMPACT IN CLUSTERING

Applying to ML: Ensure the impact of a system across protected groups is proportionate. “Group fairness.”

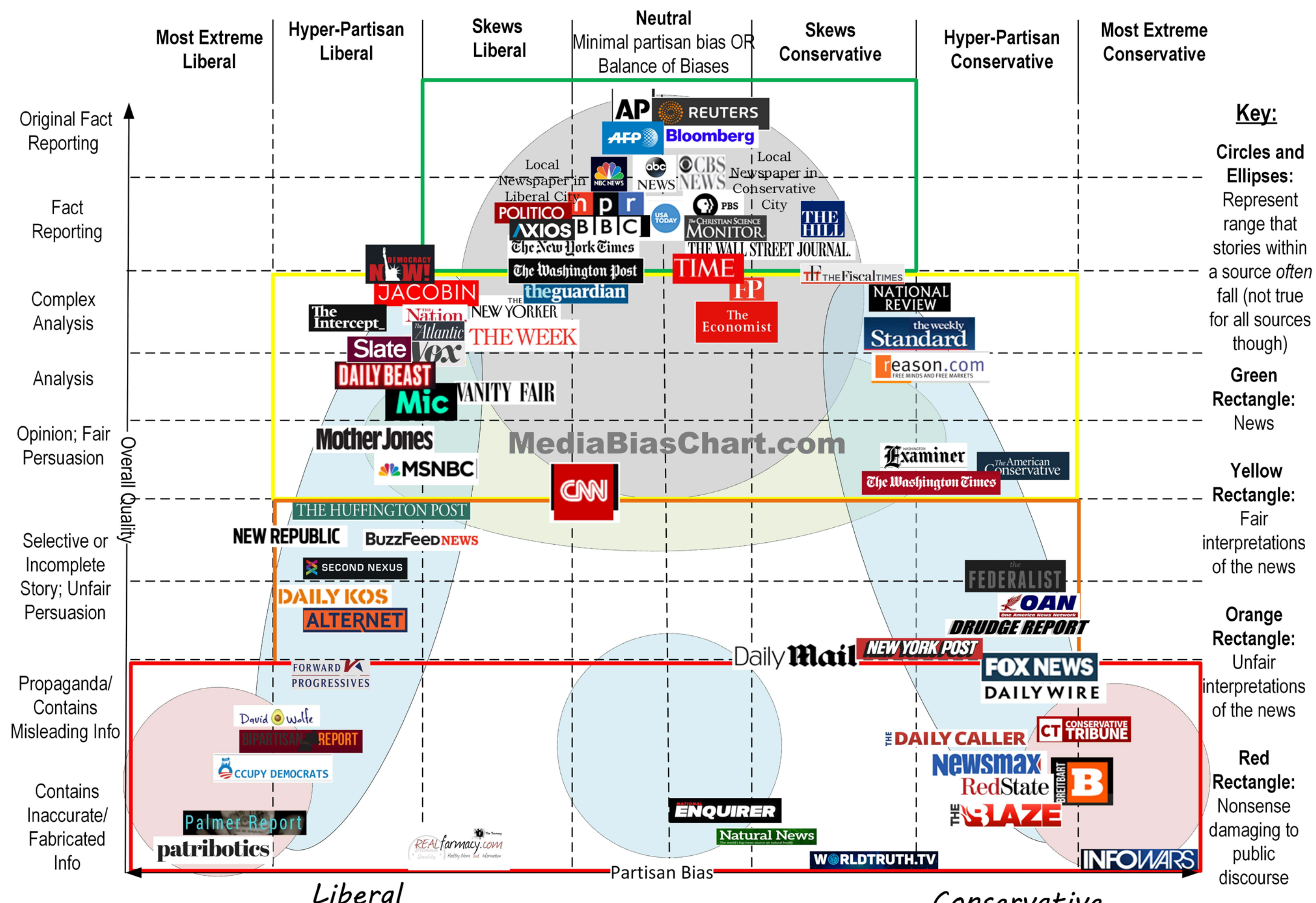
Applying to clustering:

- How do we measure the impact of a system on a protected group?
  - How many individuals are in a cluster
- How do we prevent disparate impact?
  - Ensure the number of individuals from any group in any cluster is proportionate to group size.

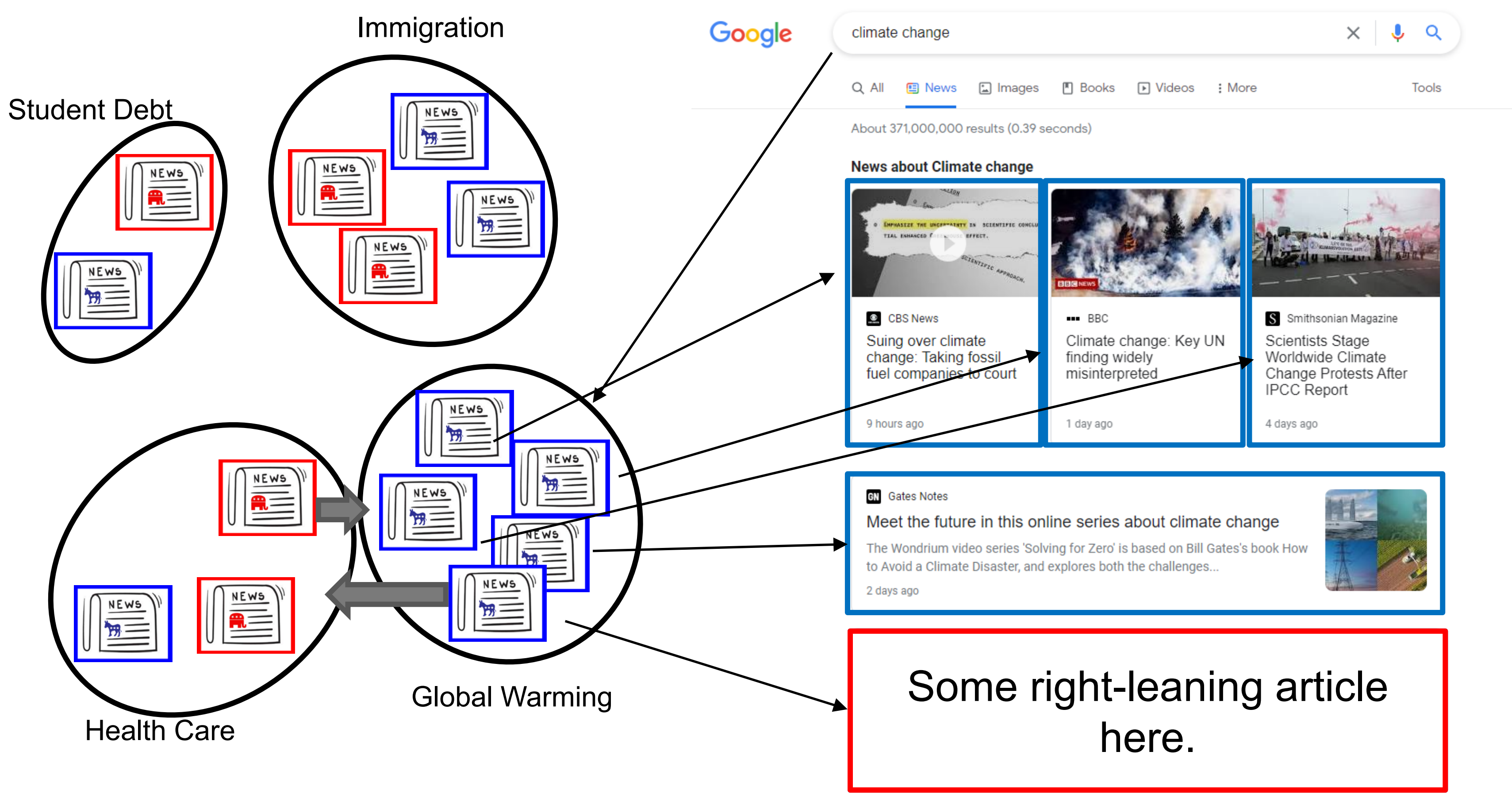
CLUSTERING NEWS ARTICLES



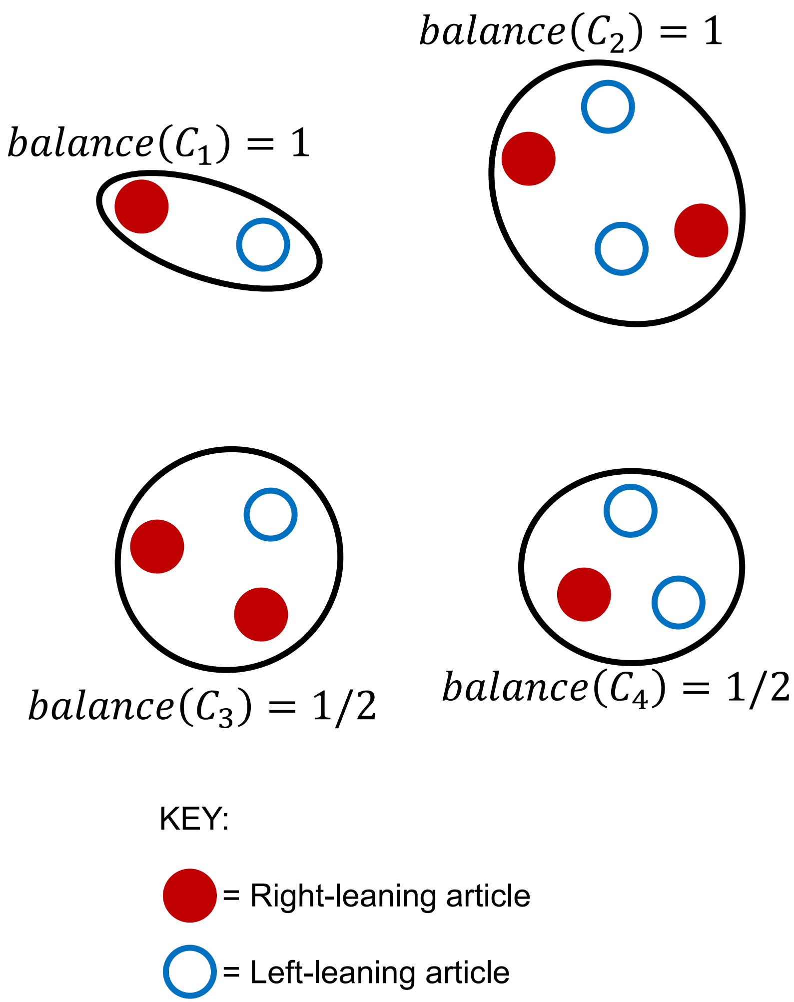
RUNNING EXAMPLE: NEWS SEARCH



CLUSTERING NEWS ARTICLES



GROUP FAIRNESS IN CLUSTERING: FORMAL



For a cluster C:

$$balance(C) \coloneqq \min\left(\frac{\#red(C)}{\#blue(C)}, \frac{\#blue(C)}{\#red(C)}\right)$$

For a clustering S:

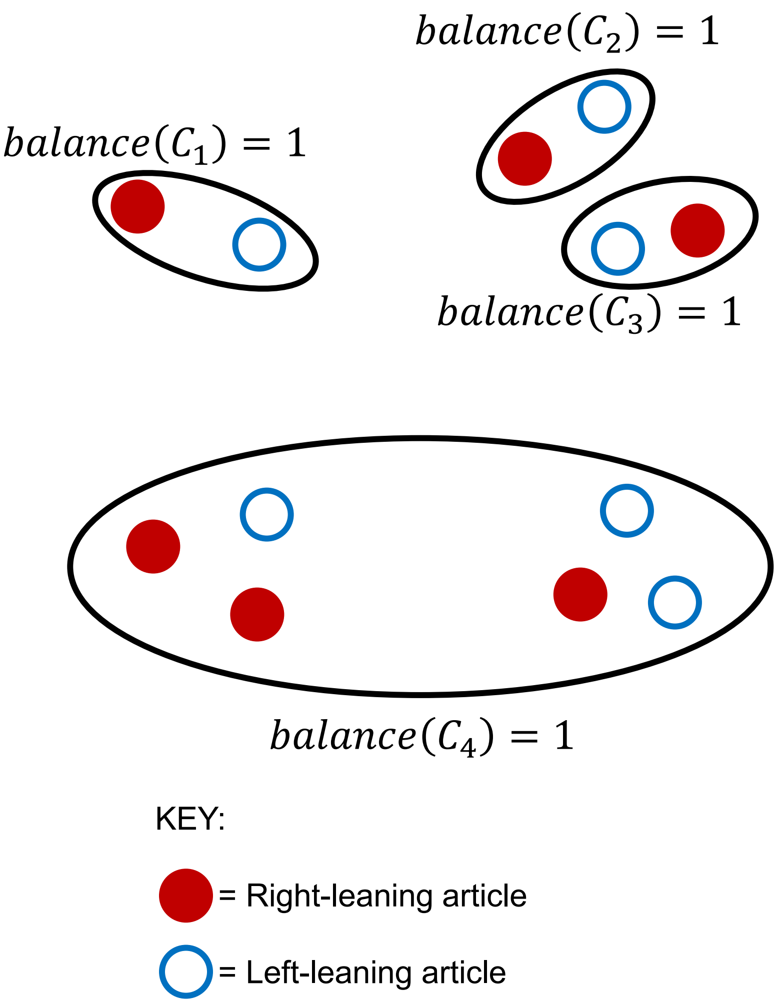
$$balance(S) \coloneqq \min_{C \in S} balance(C)$$

We want balance to be high (close to 1).

Before: $balance(S) = 1/2$

After:

GROUP FAIRNESS IN CLUSTERING: FORMAL



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For a clustering S:

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We want balance to be high (close to 1).

Before: $balance(S) = 1/2$

After: $balance(S) = 1$

FINDING FAIR CLUSTERINGS: FAIRLETS

This is the best balance we can achieve

Let:

b/r = 1/3 is the minimum ratio between reds and blues

R = number of remaining reds = 10

B = number of remaining blues = 5

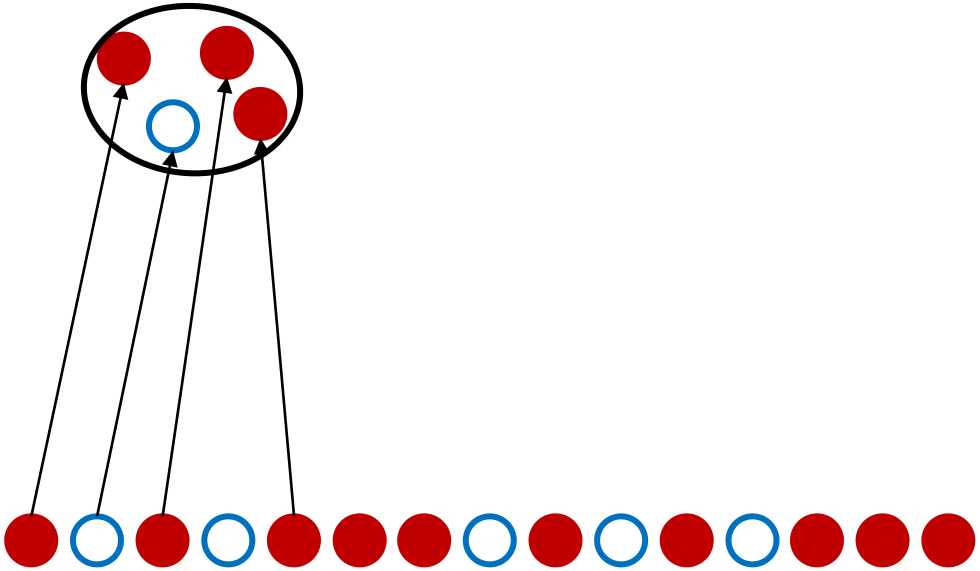
α = 1/3 is our fair parameter

Iteratively…

IF (R-B) ≥ (r-b): make a cluster of r reds and b blues.

ELSE: use (R-B)+b red and b blue points.

When r=b: simply match points



FINDING FAIR CLUSTERINGS: FAIRLETS

This is the best balance we can achieve

Let:

b/r = 1/3 is the minimum ratio between reds and blues

R = number of remaining reds = 7

B = number of remaining blues = 4

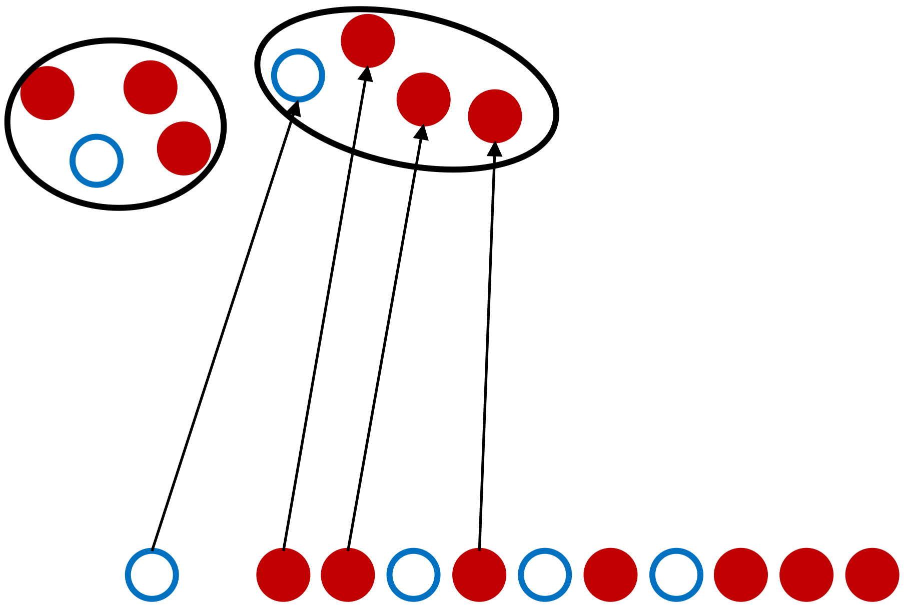
α = 1/3 is our fair parameter

Iteratively…

IF (R-B) ≥ (r-b): make a cluster of r reds and b blues.

ELSE: use (R-B)+b red and b blue points.

When r=b: simply match points



FINDING FAIR CLUSTERINGS: FAIRLETS

This is the best balance we can achieve

Let:

b/r = 1/3 is the minimum ratio between reds and blues

R = number of remaining reds = 4

B = number of remaining blues = 3

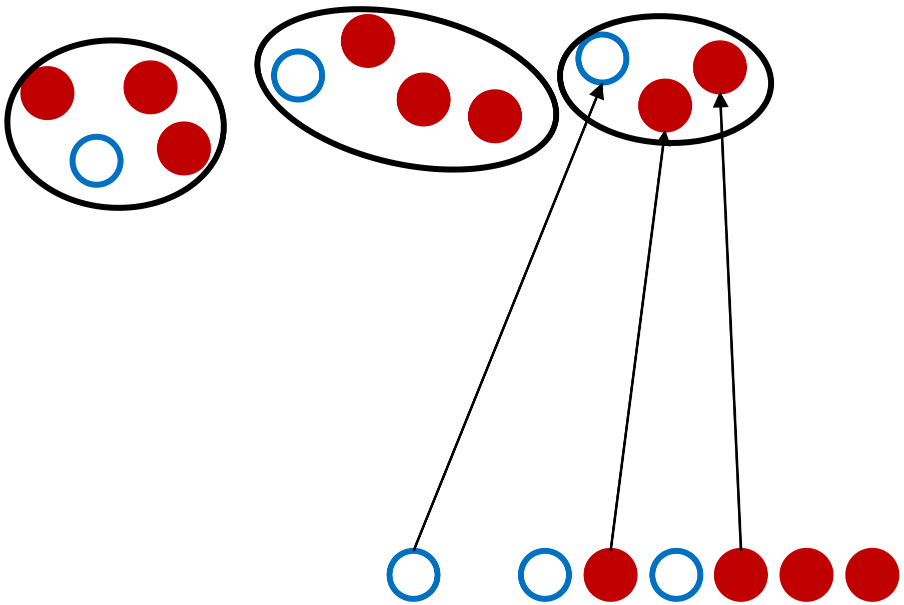
α = 1/3 is our fair parameter

Iteratively…

IF (R-B) ≥ (r-b): make a cluster of r reds and b blues.

ELSE: use (R-B)+b red and b blue points.

When r=b: simply match points



FINDING FAIR CLUSTERINGS: FAIRLETS

This is the best balance we can achieve

Let:

$b/r = 1/3$ is the minimum ratio between reds and blues

R = number of remaining reds = 2

B = number of remaining blues = 2

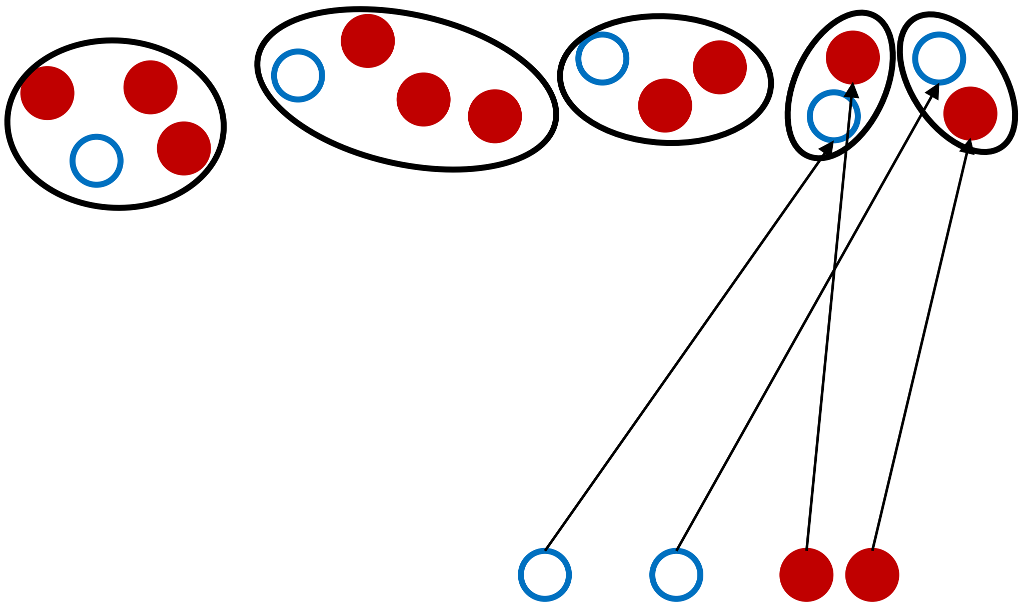
$\alpha = 1/3$ is our fair parameter

Iteratively…

IF $(R-B) \geq (r-b)$: make a cluster of r reds and b blues.

ELSE: use $(R-B)+b$ red and b blue points.

When r=b: simply match points



FINDING FAIR CLUSTERINGS: FAIRLETS

This is
the best balance
we can achieve

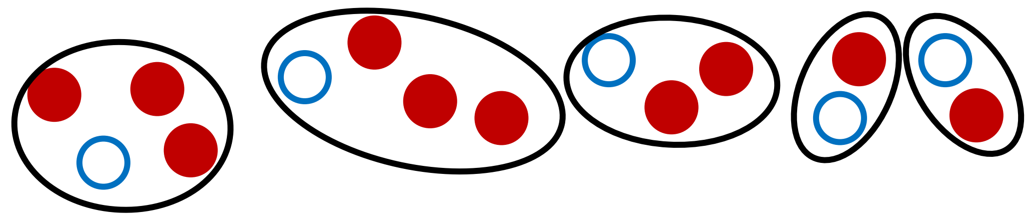
Let:

$\boxed{b/r = 1/3}$ is the minimum ratio between reds and blues

R = number of remaining reds = 0

B = number of remaining blues = 0

α = 1/3 is our fair parameter



Iteratively…

IF (R-B) ≥ (r-b): make a cluster of r reds and b blues.

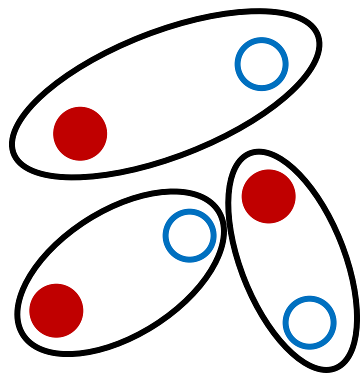
ELSE: use (R-B)+b red and b blue points.

When r=b: simply match points

Idea: Find a "fairlet decomposition" with small fairlets. Find a way to merge them into a good, appropriately-sized clustering.

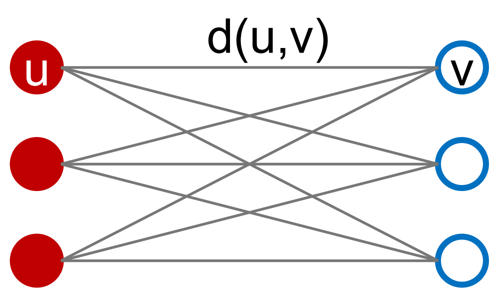
FAIRLETS WHEN B/R=1

Say half of your points are red and half are blue.



Make a bipartite graph between red and blue points. Edge weights are distances.

Find a perfect, maximum weight matching.



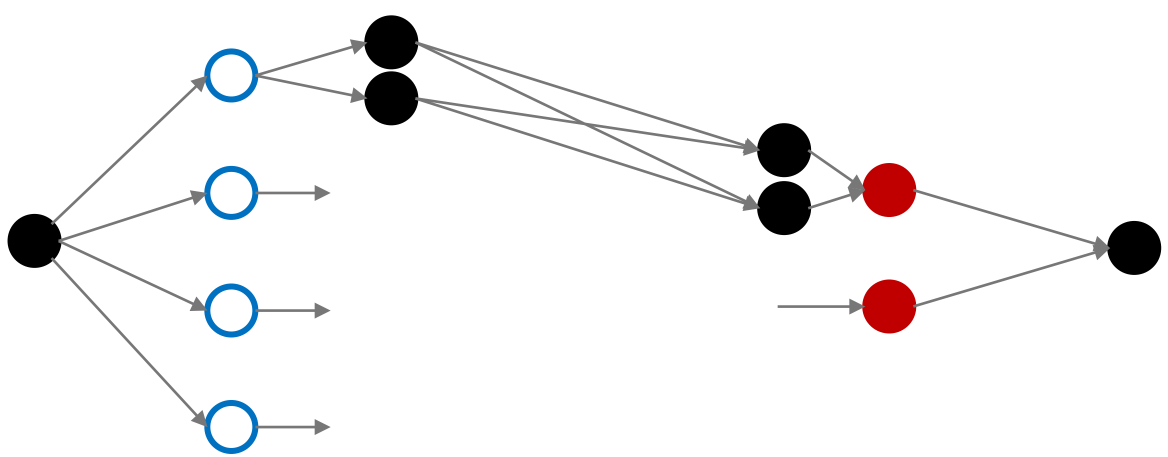
Construct 2-sized fairlets made of the matches.

FAIRLETS WHEN B/R=1/T'

Let B be the number of blues, R be the number of reds. Assume B/R = 1/t' for an integer t'.

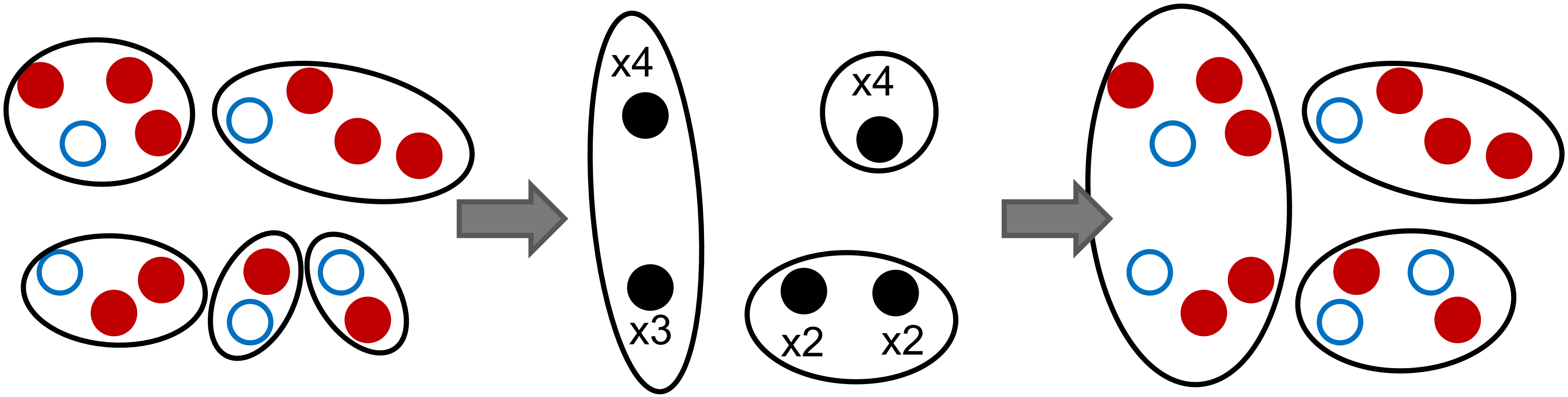
Fairlets can be found using *minimum cost flow* (a network flow problem where edges are labeled with associated edge costs, edge flow capacities, and a flow supply/demand at vertices.

t' = 2



USING FAIRLETS FOR FAIR CLUSTERING

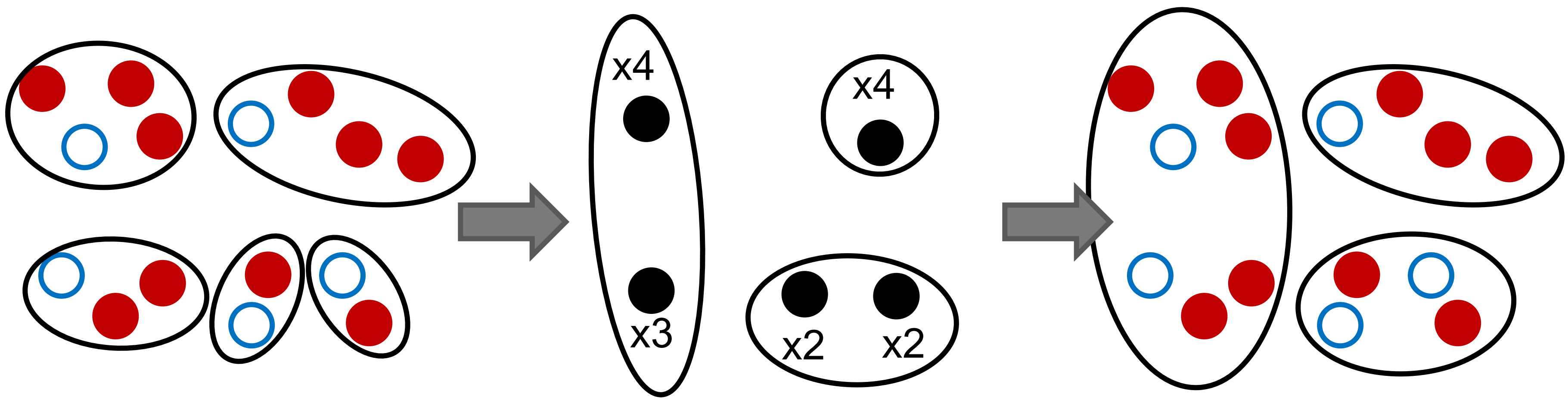
1. Find a *good* fairlet decomposition.
2. Replace each fairlet with a single point. Duplicate that point according to the fairlet size.
3. Run a “vanilla” clustering on these points.
4. Apply this clustering to the original points.



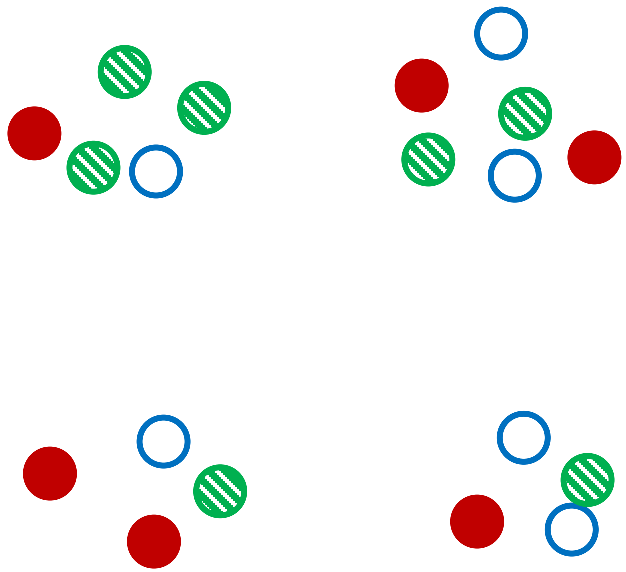
USING FAIRLETS FOR FAIR CLUSTERING

Let: *Y* be our fairlet decomposition, *Y'* be our transformed point set, and *S* be our final clustering.

Theorem: for k-median and k-center: $cost(S) = cost(Y) + cost(Y')$



GENERALIZING TO MORE COLORS



For each color *i*, we get bounds α_i, β_i to bound the proportional representation of *i*.

A cluster *C* is fair if for all colors *i*:

$$\alpha_i \times |C| \leq i(C) \leq \beta_i \times |C|$$

Where $i(C)$ is the number of i colored points in C.

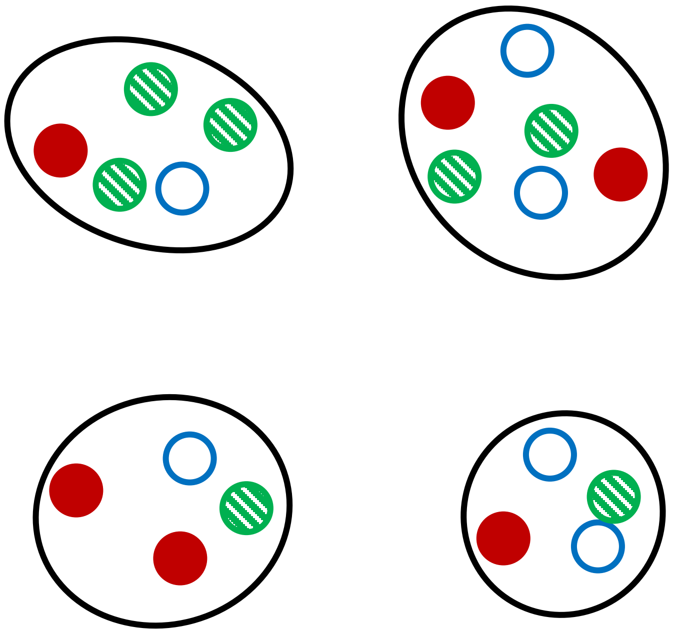
KEY:

= Right-leaning article

= Left-leaning article

= Green party-leaning article

GENERALIZING TO MORE COLORS



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Is this clustering fair for the following values?

α_{red}= 1/4 ✗ β_{red} = 1/2 ✓

α_{green}= 1/3 ✗ β_{green} = 1/2 ✗

α_{blue}= 1/4 ✓ β_{blue} = 1/2 ✓

What about…

α_{red}= 1/5 ✓ β_{red} = 1/2 ✓

α_{green}= 1/4 ✓ β_{green} = 3/5 ✓

α_{blue}= 1/4 ✓ β_{blue} = 1/2 ✓

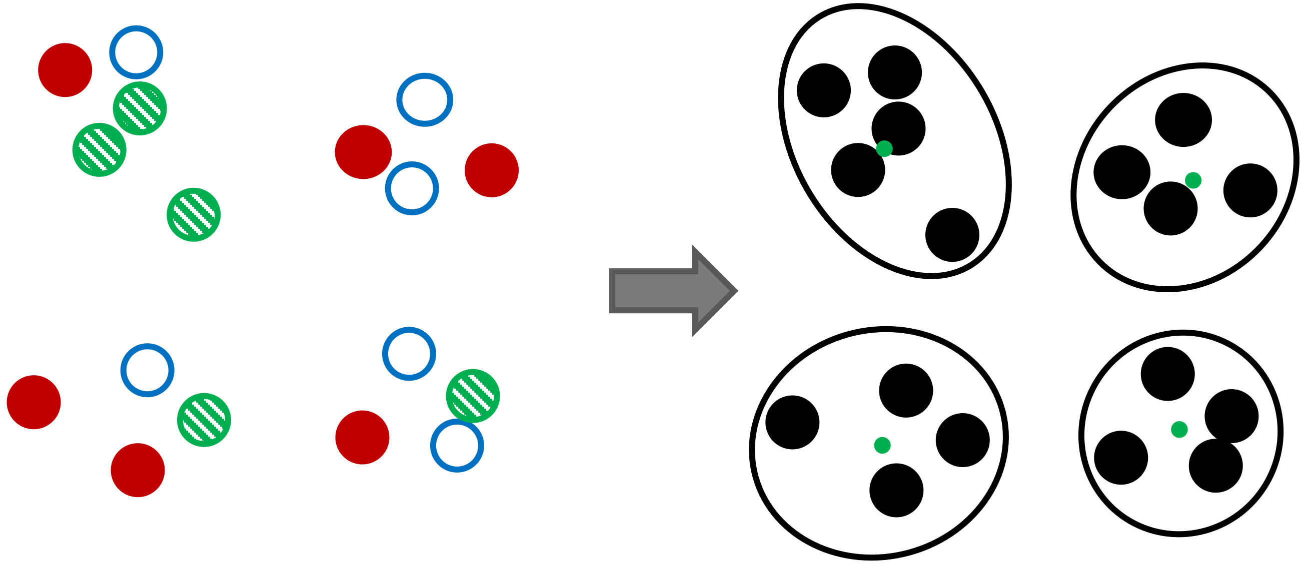
ILP FOR FAIR CLUSTERING

For this approach, we will use a linear program as an intermediate step, *but it will not be used to solve the whole problem*.

1. Ignore fairness. Find a good vanilla clustering. This gives cluster centers.
2. Use a LP relaxation of an ILP to fairly assign points to centers.
3. Round the LP to an integer solution.

ILP FOR FAIR CLUSTERING STEP 1

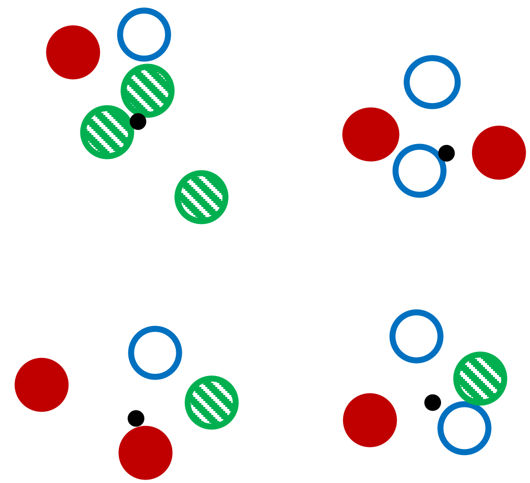
Step 1: Ignore fairness. Find a good vanilla clustering.



ILP FOR FAIR CLUSTERING STEP 2

Step 2: Use a LP relaxation of an ILP to fairly assign points to centers.

Let $C = c_1, c_2, \ldots, c_k$ be our given cluster centers.
Let x_i denote that x is assigned to center i.
Let *R* be our distance guess.



Objective: None!

Constraints: $\sum_{c_i \in C \cap B_R(x)} x_i = 1$ for all $x \in X$

$\alpha_i \sum_{x \in X} x_i \leq \sum_{j \text{ colored } x \in X} x_i \leq \beta_i \sum_{x \in X} x_i$ for all $c_i \in C$ and colors *j*

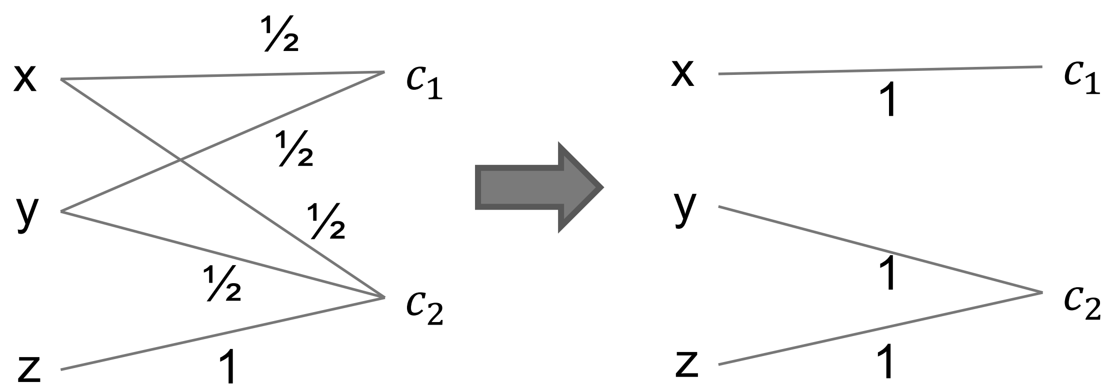
$0 \leq x_i \leq 1$ for all $x \in X$ and $c_i \in C$

ILP FOR FAIR CLUSTERING STEP 3

Now we have a set of cluster centers and a fair *fractional assignment* of points to centers.

In other words, *x* can be ½ assigned to one center and ½ assigned to another!

Step 3: Round these to whole numbers (i.e., a real assignment).



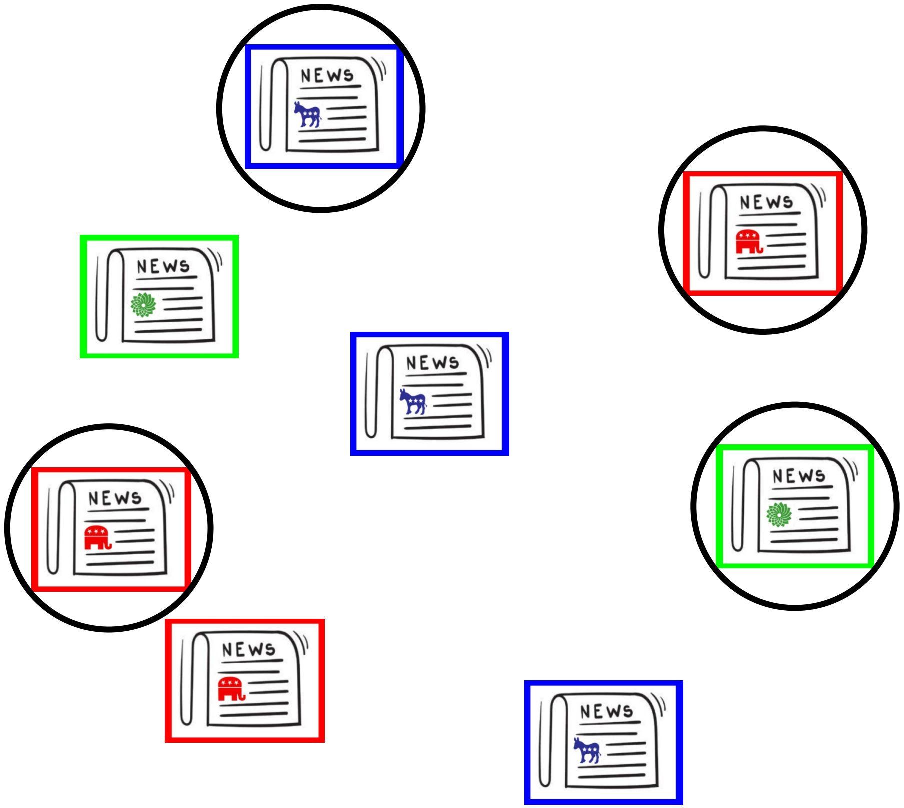
FAIR DATA SUMMARIZATION

Data summarization: Select a subset of points that "represent" your data.

Method: Do a clustering, and select the cluster centers.

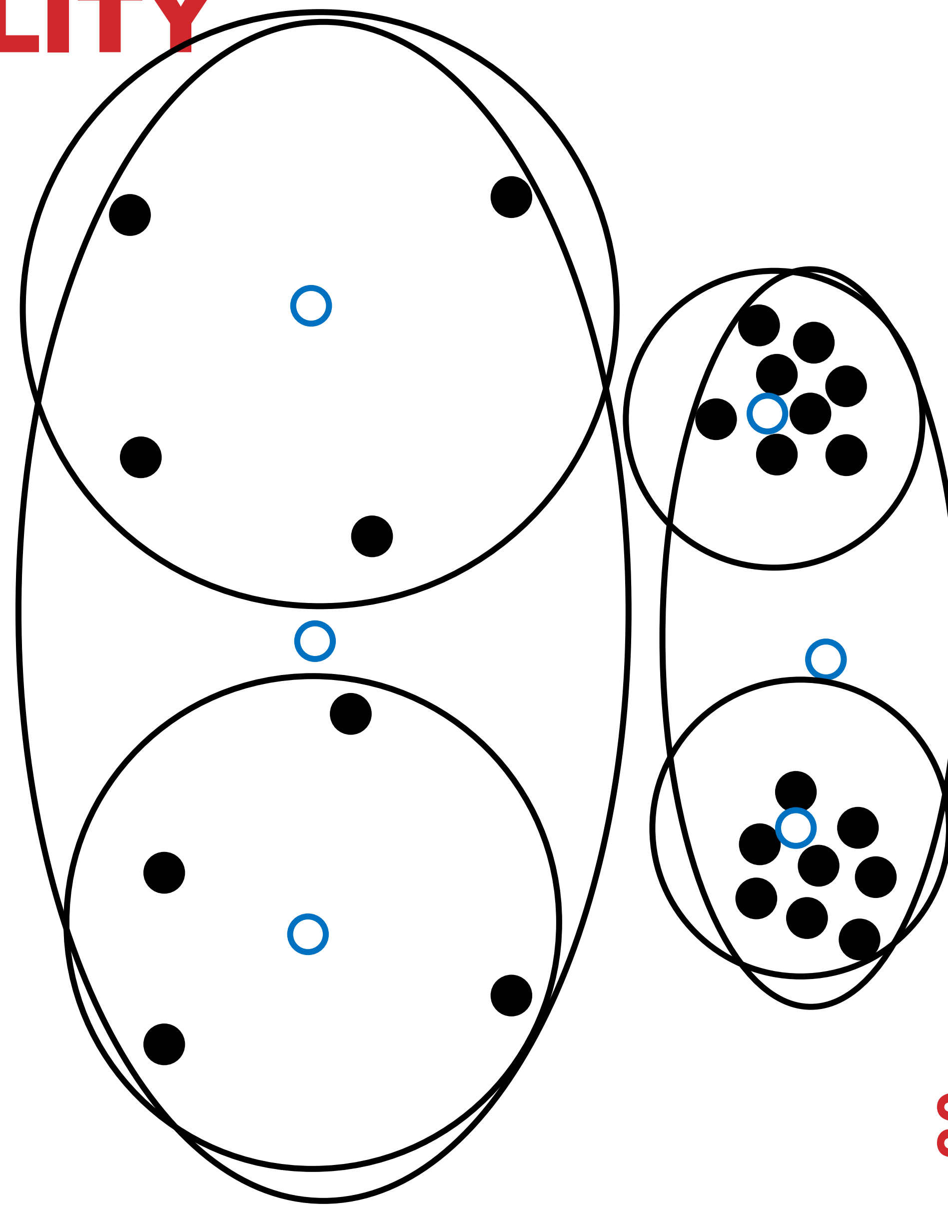
Fairness: for every color *i*, there must be at least k_i representatives of that color.

Say $k_{red} = k_{blue} = k_{\text{green}} = 1$



FAIRNESS THROUGH PROPORTIONALITY

Say you want to build parks 3 parks. You have two dense cities and 1 rural area. There are 6 park location options (blue-rimmed circles).



Blocking coalition: a set of *n/3* people such that there is 1 park location that they *all* prefer to their given location.

Proportional clustering: one with no blocking coalition.